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INFORMATION EXTRACTED FROM STRONG-MOTION RECORDS USING COMPUTER CALCULATIONS

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INTRODUCTION

Although useful information can be extracted from strong-motion earthquake records by visually examining time-history plots of the acceleration and by performing simple calculations by hand, it is now a routine procedure to extract further information by computer processing and analysis of the records. The emphasis here will be on two major tools used in this routine analysis, response spectra and Fourier amplitude spectra. In addition, a technique called system identification will be discussed which can be used in special studies of recorded motions of structures such as buildings, bridges and dams.

For computer processing, the data must be prepared in a digital form corresponding to sampled values of the acceleration measured at the instrument's location. These digitized values normally correspond to the motion sampled at equal time steps. In the case of a digital accelerograph they are produced directly by the instrument, but in analog accelerographs a special digitizing procedure must be applied to the original records which are produced as traces on film. Once the digitized data have been input to the computer, a calibration of the data is performed, followed by corrections for possible baseline distortion and for the frequency response of the recording instrument. These corrected accelerations are then in effect integrated to produce velocity and displacement time histories. The next step is usually to calculate and plot the response spectra and Fourier amplitude spectra.

RESPONSE SPECTRA

Introduction

Response spectra are a way of characterizing the earthquake ground motion in terms of its potential attack on a certain class of structures.

For a given recorded ground motion, they show in graphical form the peak induced motion in a range of simple structures illustrated in Fig. 1. Since the deformation of these structures is described by a single coordinate (x), they are referred to as single-degree-of-freedom (SDOF) structures. Their dynamic properties are described by two parameters: T , the natural period of free vibrations, and h , the damping factor controlling the rate at which energy is dissipated by the damping force. An ordinate at period T of a response spectrum corresponding to damping h gives the largest response produced in the corresponding SDOF structure by the recorded ground motion. The response may represent the displacement (x) or velocity (\dot{x}) of the mass (m) relative to the ground, or the absolute acceleration ($\ddot{x} + \ddot{z}$) of the mass relative to an inertial (fixed) reference system.

For example, each solid curve in Fig. 2 represents the peak relative velocity induced in various SDOF structures with a specified damping h and a range of periods when these structures are subjected to a component of ground motion recorded in the 1940 Imperial Valley earthquake. The top curve corresponds to no damping (0%), the next solid curve to 2% damping, then 5%, 10% and finally 20% damping. The damping h is expressed as a percentage of the critical value, which is the amount required to prevent any oscillatory motion about the equilibrium position when the system is pulled aside and released.

Computation

Let $x(t)$ denote the relative displacement at time t of a SDOF structure of period T and damping h subjected to the ground acceleration $\ddot{z}(t)$, then these quantities are related by the equation of motion:

$$\ddot{x} + 2hw\dot{x} + w^2x = -\ddot{z}(t) \quad (1)$$

with initial conditions $x(0)=0$ and $\dot{x}(0)=0$, and where $w = 2\pi/T$. Various response spectra may then be defined by:

Relative displacement spectrum:	$SD(T,h) = \max x(t) $
Relative velocity spectrum:	$SV(T,h) = \max \dot{x}(t) $
Absolute acceleration spectrum:	$SA(T,h) = \max \ddot{x}(t) + \ddot{z}(t) $
Pseudo-velocity spectrum:	$PSV(T,h) = w \cdot SD(T,h)$
Pseudo-acceleration spectrum:	$PSA(T,h) = w^2 \cdot SD(T,h)$

The "pseudo" spectra are introduced as a convenient way of representing displacement, velocity and acceleration spectra on the same plot. In this regard, it may be shown that PSV and PSA are satisfactory approximations for SV and SA respectively, with the latter approximation being more accurate than the former (Hudson, 1979). It may also be shown that in a logarithmic plot of PSV versus period T , the corresponding values of SD and PSA may be read off from axes rotated clockwise and anticlockwise respectively by 45 degrees. This is illustrated by the so-called "tripartite plot" in Fig. 3 which is for the same earthquake record as in Fig. 2.

To construct response spectra, a large number of calculations are involved in which the response $[x(t), \dot{x}(t) \text{ and } \ddot{x}(t) + \ddot{z}(t)]$ must be computed at each time step using Eq. 1 for a range of periods and damping factors. Each calculated response history is then automatically searched for the largest value to give a response spectral ordinate for that period and damping.

One complication in solving Eq. 1 is that the ground acceleration $\ddot{z}(t)$ is not given as a continuous function of time but as a series of discrete values corresponding to the sampling timestep, typically 0.01 or 0.02 second. The differentiation in Eq. 1 must therefore be done in some approximate manner. A number of computer algorithms are available to do this which are often derived by making some assumption about how either the response or the excitation \ddot{z} varies between the discrete sample times. As an example, one commonly used

algorithm makes the approximation that \ddot{z} varies linearly over each timestep (Nigam and Jennings, 1969). This particular algorithm was used to compute the response spectra of several hundred earthquake records which are presented in a series of Caltech reports (EERL, 1970-75). The introduction to Part A, Volume III, of these reports gives a summary of the algorithm, together with a brief discussion of response spectra.

Applications

One of the principal applications of response spectra are during the earthquake-resistant design of structures where they are often used to estimate the peak response which would have occurred during an earthquake. In this case, the structures are, of course, more complicated than the simple SDOF ones, but the peak may be estimated by representing the response at any point as a superposition, or sum, of contributions from modes of vibration and then using the response spectrum to determine the peak contribution of each mode during the given earthquake. This can be done because the response of a mode is just a scaled version of the response of the simple SDOF structure with the same period and damping. It turns out that only a few of the lower frequency modes contribute significantly to the earthquake response and so only these need to be considered. The method is called "mode superposition" and it is described in more detail in Chopra (1980) and Newmark and Hall (1982).

One difficulty with this approach is that the largest responses for each of the modal contributions do not occur at the same time in general (Fig. 4). The maximum response is therefore not given simply as the sum of the maximum responses of each mode. One common approach is to estimate the peak response by the square root of the sum of the squares of the maxima for each mode. For example, for the roof displacement in Fig. 4c, the response spectrum would have given 4.91, 1.56 and 0.10 inches for the peak displacement of the first three modes corresponding to the three lowest natural frequencies. The maximum roof displacement would therefore be estimated by the square root of $(4.91^2 + 1.56^2 + 0.10^2)$, which is equal to 5.15 inches compared with the 5.16 inches shown in Fig 4 as the actual maximum displacement. Similarly, the maximum base shear in Fig. 4d is seen to be 346 kips whereas the estimated peak from the maxima of the three modes would be 327 kips.

As can be seen from Figs. 2 and 3, the response spectra vary somewhat erratically, particularly for the lower damping curves. In using spectra in design, a damping of 5% is typically assumed and the spectral curve is smoothed by averaging out some of the small troughs and peaks. Sometimes many such spectra of similar intensity may be averaged to produce a design spectrum which is representative of a class of earthquake records.

Since the intensity of ground shaking at a site depends on the magnitude of the earthquake event and the distance of the site from the fault rupture, together with other factors, the response spectrum will also depend on these parameters. To allow rough predictions during design, response spectral ordinates have been correlated empirically against magnitude and distance (see, for example, McGuire, 1977). Sometimes a simpler approach is taken where the estimated peak ground displacement, velocity and acceleration are used to estimate the response spectrum (Newmark and Hall, 1982). As would be

expected, the basic behavior of the response spectrum is for the overall level to increase with increasing magnitude and to decrease with increasing distance from the source of the earthquake. In addition, the shape of the spectrum changes with distance because the high frequency components of the seismic waves are attenuated more rapidly as they propagate through the Earth's crust. The shape also changes with magnitude because an increase in magnitude puts more energy into the low frequency waves than those at higher frequencies.

As a final comment, response spectra for structural motions are also sometimes computed, although their primary use relates to ground motions. For building motions, they can give "floor spectra" which are used to estimate how severely equipment located on that floor would be excited during the earthquake.

FOURIER SPECTRA

Introduction

The Fourier spectrum is a means of analyzing the frequency content of the ground or structural motion at a particular location.

This analysis is achieved by considering the motion as a superposition, or combination, of pure harmonic (sinusoidal) waves. Usually only the amplitude of each harmonic component is plotted against its frequency or its period, and this produces the Fourier amplitude spectrum (FAS) of the recorded motion. For example, see the dashed curve labelled FS in Fig. 2, which is the FAS of a component of the ground acceleration recorded in the 1940 Imperial Valley earthquake. A Fourier phase spectrum may also be plotted against frequency giving a phase in degrees (or radians) which may be related to the time-shift of the harmonic component at that frequency relative to a reference harmonic wave with the same frequency and amplitude but which starts from zero at time $t=0$. This interpretation can be deduced from the mathematical definition below.

Computation

Let $\ddot{z}(t)$ denote the recorded acceleration, then expressing this as a superposition of n harmonics:

$$\ddot{z}(t) = a_0 + a_1 \sin(w_1 t + e_1) + a_2 \sin(w_2 t + e_2) + \dots + a_n \sin(w_n t + e_n) \quad (2)$$

where $w_k = 2\pi f_k$, and $f_k = k/TD$, $k=1,2,\dots,n$. Here f_k is the frequency of the k th harmonic and TD is the duration (length in seconds) of the recorded time history $\ddot{z}(t)$. The amplitude and phase coefficients, a_k and e_k , $k=1,2,\dots,n$, give the Fourier amplitude and phase spectra respectively when plotted against their frequency f_k . Usually the values at the discrete frequencies are connected by straight line segments to produce a continuous curve.

For a continuous time history $\ddot{z}(t)$, the coefficients a_k and e_k are given by integrals involving $\ddot{z}(t)$ over its duration TD which may be found in Brigham (1974). The coefficient a_0 , the zero frequency or "DC component", is simply the average value of acceleration over its duration. Since the full time history of the acceleration is not available, the integrals are approximated by

sums involving \ddot{z} at each of the sampling times. A computationally efficient method is available for doing these calculations called the Fast Fourier Transform (FFT) algorithm. One important consequence of having available only the sampled values at timesteps of DT is that the original spectrum of the complete acceleration history is "aliased". This means that all the information about harmonics with frequencies greater than the Nyquist frequency $[=1/(2xDT)]$ is lost and, what is potentially more serious, that their contributions get confused with those at lower frequencies in the spectrum. Fortunately, aliasing can be made small by taking the sampling interval DT sufficiently small. These points and the FFT algorithm are discussed in more detail in Brigham (1974). Part A, Volume IV, of the Caltech strong-motion reports (EERL, 1970-1975) also gives a brief coverage.

One detail which should be noted is that the Fourier spectra are often defined in such a way that an integral transform rather than the above Fourier series is used. However, since finite-duration discrete-time data are used in the calculations, the series or sum expansion is appropriate and is easier to interpret. To produce consistency with the integral representation the amplitudes a_k above should be multiplied by the duration TD , which has in effect been done in Fig. 2.

Finally, it can be observed from Fig. 2 that the curve for the FAS lies below that for the undamped relative velocity spectrum, SV. That this should be so can be supported by a simple argument. Briefly, it can be shown that the FAS of an acceleration record at period T is equal to the maximum velocity after the earthquake of an undamped SDOF structure of natural period T , whereas SV is the maximum velocity during the entire motion of the same simple structure. The FAS and undamped SV curves coincide in Fig. 2 whenever the maximum velocity of the entire motion occurs after the earthquake has ended. A similar argument shows that FAS spectrum lies below the undamped pseudo-velocity spectrum, PSV.

Applications

The FAS is a useful tool whenever there is interest in how the energy of the motion is distributed among the different frequency components or harmonics. It may be used, for example, to study the effects of local site conditions on the ground motion if records from a local array of instruments are available. It is also used in seismological studies of the earthquake source mechanisms and of the changes in the seismic waves produced during propagation through the Earth's crust. As a final example, the modes of vibration produce "resonant" peaks in the FAS of structural motions which may be used to estimate the natural frequencies of those modes which are strongly excited by the earthquake.

SYSTEM IDENTIFICATION

Introduction

In the present context, system identification refers to systematic methods of determining mathematical models of a structure from its measured earthquake excitation and response. The applications of system identification to strong motion records have been surveyed by Beck (1982).

One method which has been used successfully to determine the natural frequencies and damping factors of those modes of vibration of an instrumented structure which were strongly excited by an earthquake is implemented as follows. The parameters are estimated by using an iterative optimization algorithm to find those values which produce the best agreement possible between the measured response and that of the model when it is subjected to the measured excitation. The agreement is quantified by a measure-of-fit J which may be taken to be the sum of the squared errors between the measured and model responses at the discrete sampling times.

Computation

Let $\underline{\theta}$ denote a vector of unknown parameters of the structural model such as the natural periods and dampings of the modes of vibration. They could also be more basic member properties such as Young's modulus and moments of inertia but unless there are only few such parameters, problems with uniqueness may arise. For example, there may be a whole range of values of the parameters which produce the same model response to the measured excitation. The question of uniqueness involves a study of the identifiability of the models (Beck, 1978 and Udwadia and Sharma, 1978).

Let $a(t_n)$ denote the measured acceleration at the discrete sampling times $t_n = nDT$, $n = 0, 1, \dots, N$, and let $\ddot{x}(t_n; \underline{\theta})$ be the corresponding acceleration calculated from the model for the prescribed values of the parameters and for the measured excitation. The measure-of-fit between the measured and model accelerations is then defined by:

$$J(\underline{\theta}) = \sum_{n=0}^N [a(t_n) - \ddot{x}(t_n; \underline{\theta})]^2 \quad (3)$$

The parameters are estimated by the values which give J its smallest value. This means they produce the best match possible in a least-squares sense between the model and the measurements.

Various computer algorithms are available for minimizing J as a function of $\underline{\theta}$ to produce the best estimates of the parameters. One method which has proved reliable is discussed in Beck and Jennings (1980). Another related method in which J represents the match at discrete frequencies of the full complex-valued Fourier spectra of the measured and model accelerations is described in McVerry (1980).

Applications

Of course, system identification can only be applied after a structure has been built and its motions during an earthquake have been measured. An important problem in the design of structures is to be able to predict how they will behave during earthquakes. The models derived from system identification may be used to assess current practice in developing dynamic models for response prediction during design.

Since linear models based on the lower-frequency modes of vibration are commonly used in dynamic earthquake-resistant design, system identification based on linear models can provide valuable information relating to the design

process, particularly in relation to the following questions:

- 1) How good are linear dynamic models for predicting the seismic response of structures, and when do they break down?
- 2) How accurate are the parameters in the analytical models determined from structural properties in dynamic design? In particular, how do the values of the natural periods exhibited by a structure in an earthquake compare with the values used in its design?
- 3) What are appropriate values for the damping factors of the modes of vibration since these are very difficult to determine theoretically? System identification can provide typical values for a class of structures, such as reinforced-concrete frame buildings, which may then be used in design of such structures.

Based on studies of a dozen tall buildings using system identification, it appears that linear models work well, even for large amplitude motion, provided structural damage does not occur (Beck, 1982). Also, the periods estimated from nondamaging earthquake response for the lower modes are typically within 10% to 15% of the values from analytical models used in the design, while the damping factors usually lie in the range 3% to 8% of critical damping for both reinforced-concrete and steel frame buildings.

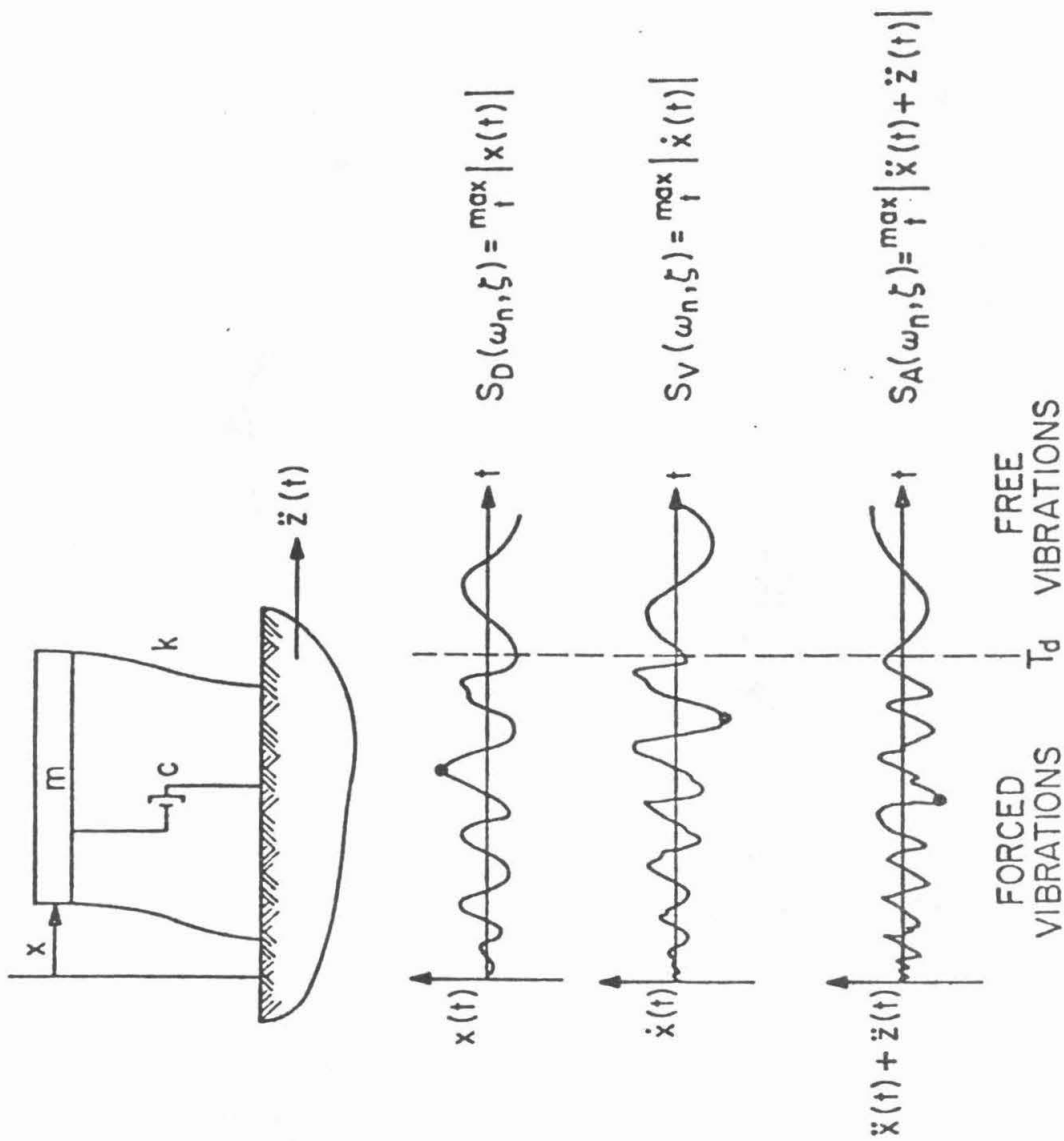
As an example, Fig. 5 shows the good agreement achieved between the displacement, velocity and acceleration at the roof of a nine-story steel-frame building during the 1971 San Fernando earthquake and the corresponding response calculated from the "best" linear model based on three modes of vibration. The parameters for this model were estimated by a least-squares match of the measured and model accelerations, and the values are shown in Table 1. The main value of the standard errors is that they demonstrate that the periods are specified much more precisely by the earthquake data than are the damping and participation factors.

The standard errors in Table 1 should not be interpreted in terms of accuracy since there are no true or exact values of the parameters to make this meaningful, in contrast to some applications of statistics in other areas. In fact, the best estimates depend on the strength of the response to a greater extent than a simple interpretation of the standard errors might imply. This is because a linear model is used to match what is actually nonlinear behavior. However, for a particular level of response the models work well as long as there is no structural damage. The dependence of the estimates on the strength of the response emphasizes the advantage of using strong-motion seismic data if available to investigate the dynamic properties of a structure, rather than experimental tests at low amplitudes. The estimates from strong-motion data will be more pertinent to the problem of earthquake-resistant design.

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RESPONSE SPECTRA

Figure 1

RELATIVE VELOCITY RESPONSE SPECTRUM

IMPERIAL VALLEY EARTHQUAKE MAY 18, 1940 - 2037 PST

IIIIA001 40.001.0 EL CENTRO SITE IMPERIAL VALLEY IRRIGATION DISTRICT COMP 500E

DAMPING VALUES ARE 0, 2, 5, 10 AND 20 PERCENT OF CRITICAL

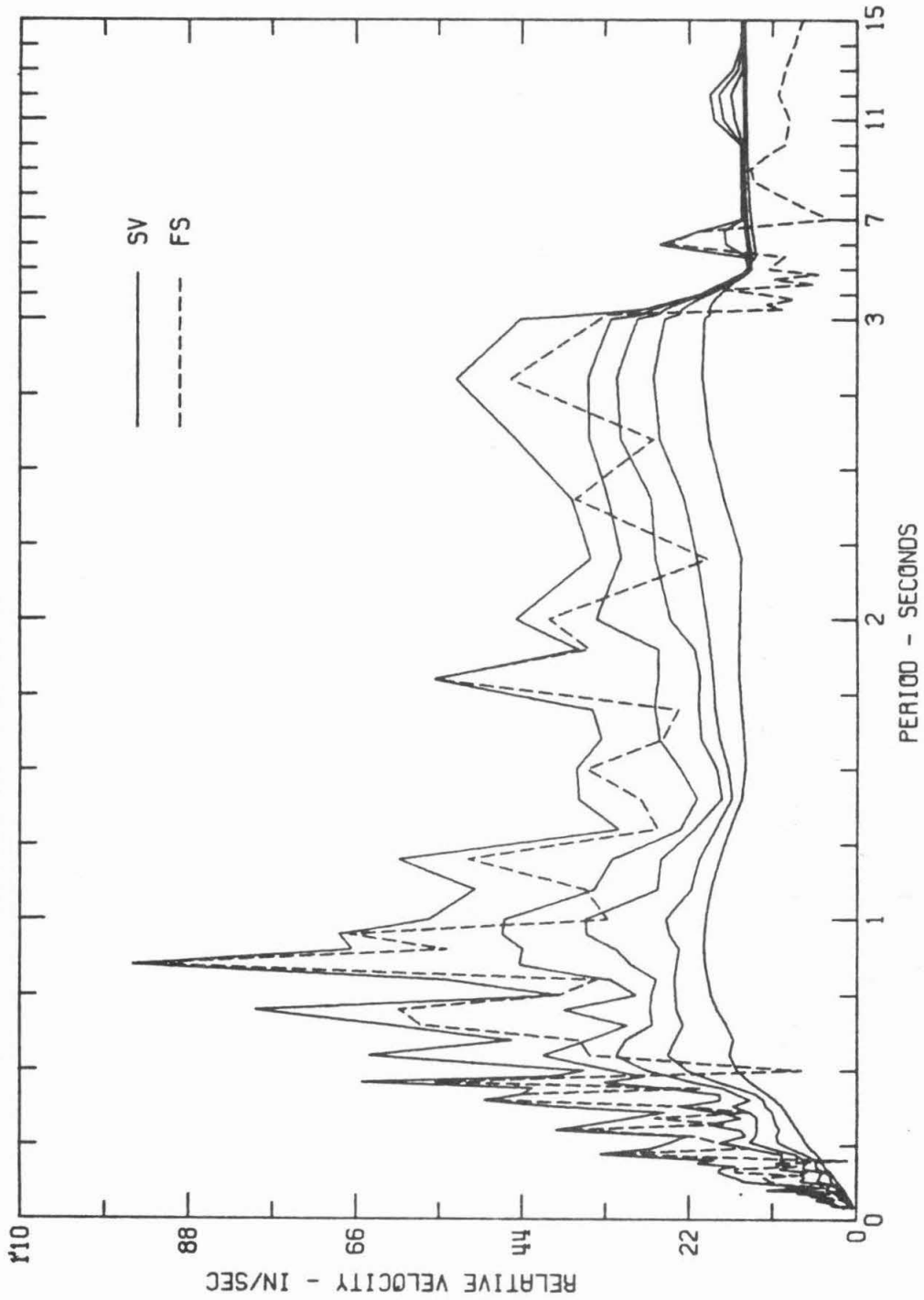


Figure 2

RESPONSE SPECTRUM

IMPERIAL VALLEY EARTHQUAKE MAY 18, 1940 - 2037 PST

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DAMPING VALUES ARE 0, 2, 5, 10 AND 20 PERCENT OF CRITICAL

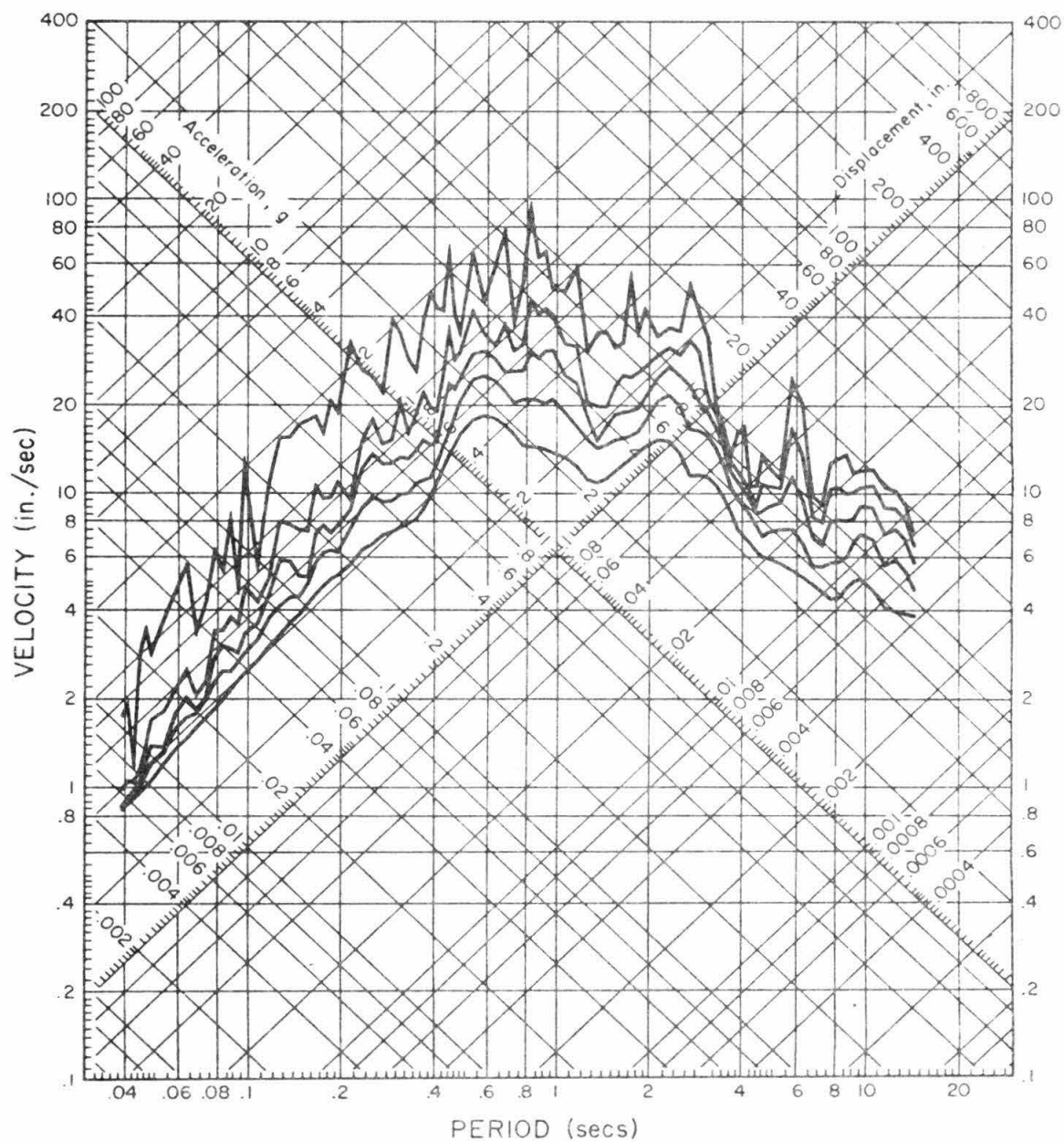
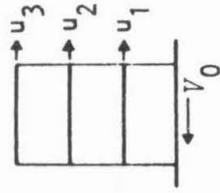
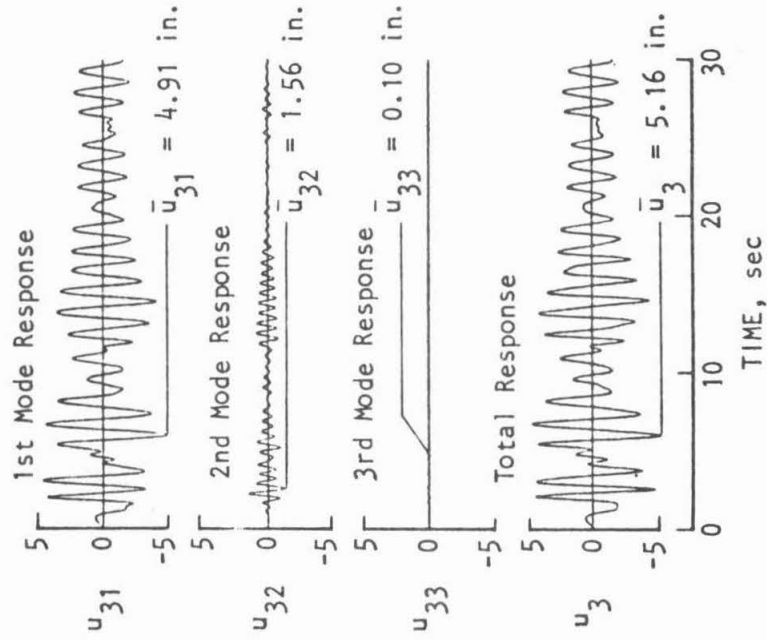


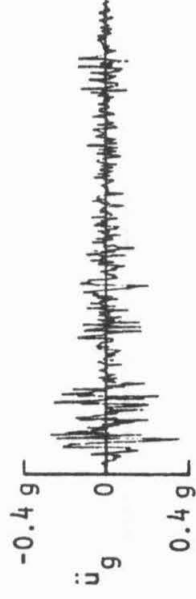
Figure 3



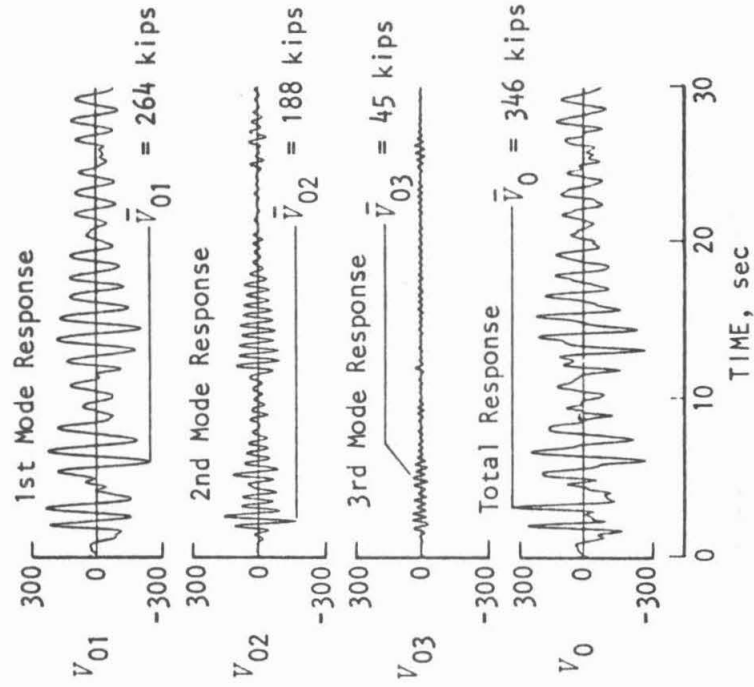
(a) Idealized three-story building



(c) Roof displacement



(b) El Centro ground motion — S00E Component
May 18, 1940



(d) Base shear

Figure 4. Earthquake response of a three-story building
(from Chopra, 1980).

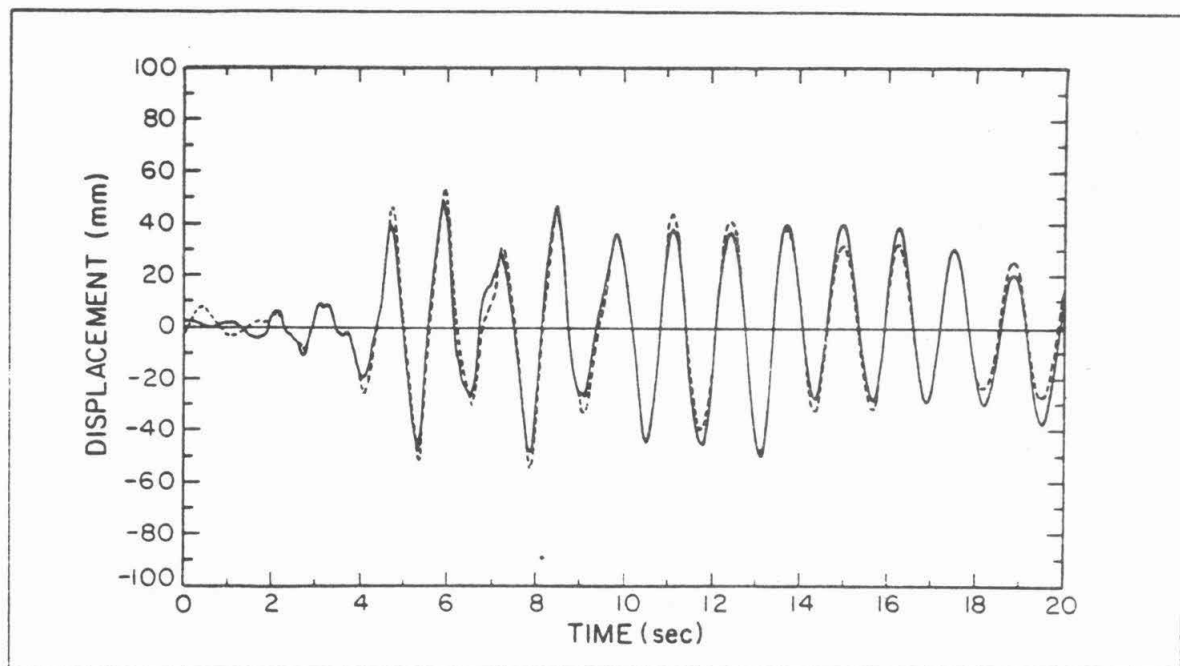


Fig. 5a. Comparison of the measured (—) and optimal 3-mode model (---) relative displacements, S82E component. The model was identified from the optimally synchronized acceleration records.

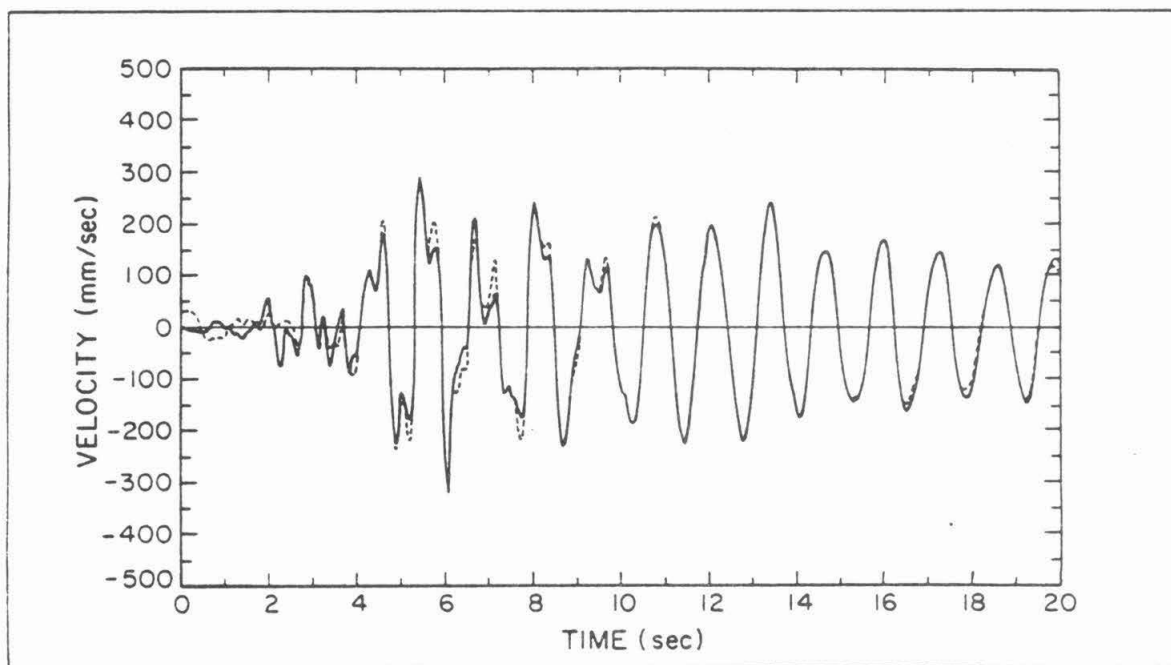


Fig. 5b. Comparison of the measured (—) and optimal 3-mode model (---) relative velocities, S82E component. The model was identified from the optimally synchronized acceleration records.

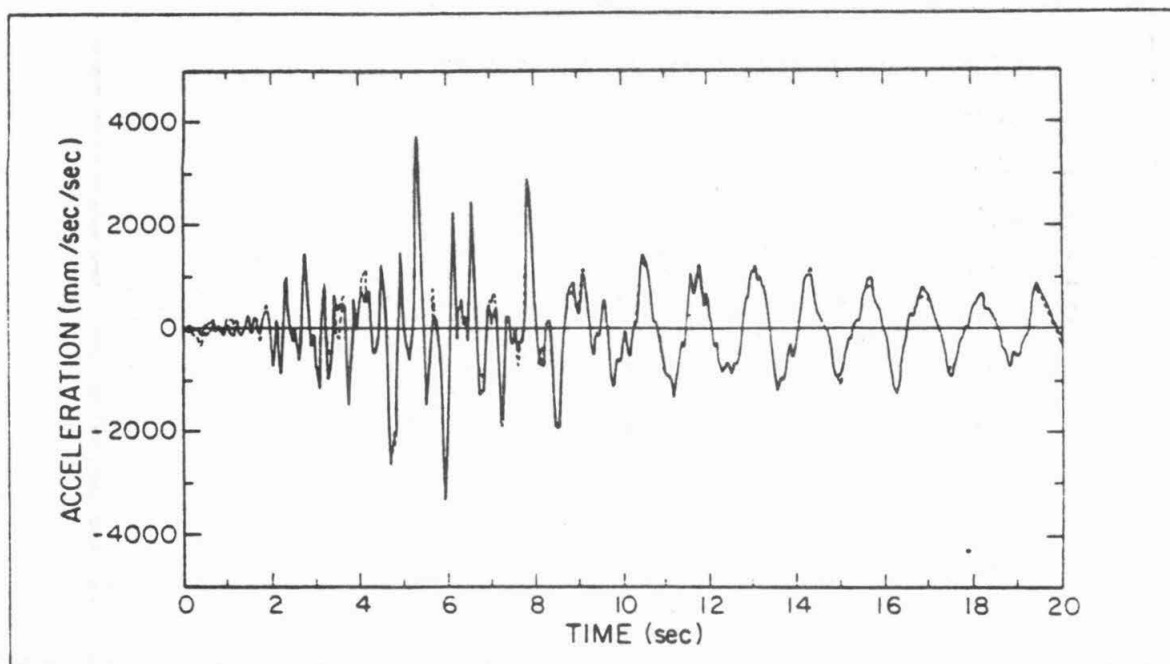


Fig. 5c. Comparison of the measured (—) and optimal 3-mode model (---) relative accelerations, S82E component. The model was identified from the optimally synchronized acceleration records.

TABLE 1

OPTIMAL ESTIMATES FROM SYNCHRONIZED
SAN FERNANDO EARTHQUAKE RECORDS, S82E COMP.,
JPL BUILDING 180

MODE	T_r	ζ_r	P_r
1	1.2765 ± 0.0006	3.56 ± 0.05	1.21 ± 0.01
2	0.4142 ± 0.0004	7.50 ± 0.10	-0.465 ± 0.005
3	0.254 ± 0.002	13.4 ± 0.6	0.28 ± 0.01
4	0.177 ± 0.002	5.0 ± 1.3	-0.07 ± 0.01